CERTAIN ASPECTS OF THE FLOW MECHANICS OF GRANULAR MEDIA. REVIEW

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Granular media have been utilized extensively in the past decade, in various branches of the national economy, as fuels, raw materials, intermediate products, catalysts, etc. Their widespread application in industry depends upon the availability of well-designed equipment that will assure the necessary reliability of automated continuous production lines. These requirements can only be satisfied when the processes that characterize granular media have been qualitatively and quantitatively determined.

Despite the extensive utilization of granular media in industry, their properties and the laws governing the fundamental processes have not yet been determined uniquely, and there is no single opinion on the basic issues. All this makes generalization of the accumulated experience difficult and delays the establishment of a scientifically grounded branch of science, the mechanics of granular media, for which there is an obvious need.

We have made an attempt to expound the principal viewpoints on individual aspects of the flow mechanics of granular media, to acquaint the reader with the most important literature, and to unite the efforts of researchers in the solution of this important problem.

The representation of a granular medium as a continuum [1–5] has stubbornly been retained in the specialized literature on the mechanics of granular media published within the past 20 years.

Although this representation can be applied to stationary granular media, such as soils, and soil mechanics has been developed successfully as continuum mechanics, it is completely inapplicable to moving media in machinery and containers.

The mechanical properties of industrial bulk solids differ fundamentally from the properties of soils. Hence, attempts to extend the quantitative relations obtained for continua (soils) at rest to bulk solid flows result in serious errors and consequently cannot satisfy the demands of practice.

Industrial granular media can be separated into two main groups, conventionally called "ideal" and "bound" (or cohesive). We agree to define ideal media as those in which there is no, or very little, cohesion between particles and dry friction is the main constraint. In contrast to ideal media, "bound" media are characterized by various cohesive constraints, as well as by friction, and these may be of comparable or even greater importance than the friction constraints themselves.

So far most research in the area of the flow mechanics of granular media has been devoted to ideal media. The mechanics of cohesive media have barely been investigated because of their complexity. Hence, we will concentrate on problems pertaining to the first group, the flow of an ideal granular medium.

We shall understand an "ideal" granular medium to be one consisting of a set of solid particles of different shape, size, and composition, whose stability is determined by the constraints of friction and uniaxial compression at particle contacts [6]. Such a definition assumes the existence of mechanical properties dependent on the friction parameters and the number of particle contacts. The latter is a function of the packing density of the bulk mixture, i.e., the porosity, which determines the number of contacts and hence the nature of the constraints imposed on the particles [7, 8].

It is desirable to use the density ratio

$$K = \gamma_b/\gamma_s$$

(γ_b and γ_s are the bulk density and solid density of the granular medium, respectively) to characterize the packing density rather than the porosity ε or the voidage fraction m.

The relation

$$K = 1 - m = \varepsilon/(1+\varepsilon)$$

exists between the density ratio and the voidage fraction.

In statics K varies between the limits K_{min} and K_{max} depending on the conditions of formation of the granular media. Under flow conditions the packing density takes a "critical" value K_{cr} between K_{min} and K_{max} [9] some time after the beginning of motion, irrespective of the initial packing density:

$$K_{min} < K_{cr} < K_{max}.$$

For granular media flowing in vertical cylindrical vessels the packing density can be assumed constant, equal to K_{cr}, over horizontal cross sections. Over the vessel height the packing density depends on the vessel diameter (D) and the thickness of the layer (h) above the section under consideration, i.e.,

$$K_{cr} = \Phi(D, h).$$

Tests have shown [9] that K depends on the surface state and size of the particles. As the particle friction coefficient increases, value of K falls markedly. The latter determines such very important parameters of a bulk solid as the coefficient of internal friction [10, 11, 12]. Thus, the angle of repose, customarily taken as the angle of internal friction, is a particular value of the angle of internal friction corresponding to that for the surface layer of a granular medium.

Generalization of the experimental data has made it possible to establish the nature, in the first approximation, of the relationship between the internal friction coefficient of a granular medium and the packing density of the particles:

$$f = \operatorname{tg}\varphi + \alpha\frac{K - K_{\min}}{K_{\max} - K},$$

where φ is the angle of repose, K the actual packing density of the granular medium, and α a constant.

External friction in granular media is the process taking place at the interface between the medium and the wall. This phenomenon is quite complex; so far its nature and quantitative behavior have not been investigated. Some authors [3] propose to characterize this process by analogy with viscous fluids in terms of internal friction, which can hardly be correct since the properties of the media are noncomparable.

One of the fundamental properties of bulk solid flows is their capacity to transmit pressure according to laws determined by the mechanism of load transmission by granular media. Numerous researchers have examined this property. Some of them extend the stress state conditions for a granular continuum to a bulk solid [2, 3, 13], others indicate its capacity to transmit horizontal and vertical pressures [1, 14]. Finally, some authors [15, 16] suggest that the flow structure is "arched" and that the load transmission mechanism is similar to that of an arch or vault.

Recent research on a great deal of experimental material [6, 11, 16] confirms the hypothesis of this last group.

In studying the kinematics of bulk solid flows, most attention has been paid to the escape of granular media from orifices in vessels. All but a few researchers assert that the discharge rate is independent of the height of the layer of material above the orifice. The qualitative side of the discharge process is variously treated. Thus, a number of researchers start by assuming the formation above the orifice of a unique structure similar to a "dynamic" arch [17–21, 24]. On the basis of this hypothesis they propose discharge formulas in which the basic parameter is the size and shape of the orifice. Others [22, 23] postulate the formation above the orifice of an "ellipsoid of motion" whose parameters are determined by the properties of the granular medium. There are authors who assert that the discharge rate does depend on the height of the layer.

However, in the past ten years research has confirmed that the discharge rate is independent of the height of the overlying layer; this is attributed to the presence of an "arched" structure above the orifice. On this basis a number of recommendations permitting substantial changes in discharge rate for a fixed orifice have been worked out and tested.

The motion of granular media flowing in pipes and troughs has received relatively little attention in the literature.

In relation to the motion of granular media in inclined pipes and chutes, the majority of authors [32–34] indicate the presence of two modes of motion: "constrained," in which the constraints imposed on the particles are not disturbed, and consequently there is no relative motion of the particles, and "unconstrained," in which the constraints are disturbed and clearly expressed relative motion is observed in the flow. The boundary between these modes corresponds to angle of inclination of the pipe (chute) to the horizontal equal to the maximum value of the angle of repose.

In relation to the motion of granular media in vertical cylindrical vessels [35–39, 41, 42, 11], all researchers also note the presence of the same two modes. Under certain conditions, one mode may go over into the other; the particle packing density is the governing factor here.

A number of authors closely connect the modes of flow of a granular medium to its mechanical properties and also note the presence of pulsating motion in the constrained mode [44].

The laws of motion of granular media in vessels when forced to flow around various obstacles placed in their path have been almost totally neglected, though individual papers [11, 43, 44, 65–68] describe flow around bodies of different geometry and determine the boundaries of flow discontinuity due to the presence of the structural elements in the flow. However, all these papers are concerned with specific effects and the conclusions are particular ones.

A majority of papers have been devoted to the pressure exerted by a granular flow on the walls and bottoms of containers. Since the second half of the last century this question has attracted the attention of many scientists. The various approaches can be grouped as follows according to the mechanism of pressure transmission in the granular medium.

The first group of authors [45, 46, 1] investigates the pressure exerted by granular media flowing in vessels by analogy with the earth pressure on retaining walls and on this basis recommends a method of determining the value of the pressure. However, this method has not been used in practice, despite the fact that it still has its supporters [46].

The second group [14, 1, 3, 47, 54–56, 64], by considering the equilibrium of a layer of granular medium and assuming the ratio of the horizontal and vertical pressures in the layer to be constant, derives working equations which are, with additional corrections [53, 57, 64], widely applied in practice. These equations are deprived of physical meaning by the corrections introduced and acquire the character of empirical formulas.

The third group [15, 40, 51] considers the mechanism of pressure transmission to be similar to that of an arch. This approach most fully reflects the physical essence of the phenomenon.

Investigation [4] in which a flow of granular medium is treated as a continuum is of undoubted interest. Starting from the theory of plasticity, and applying its apparatus, the author derives analytical equations for the pressure. However, they have not been confirmed in practice.

It should be noted that all the recommended formulas for determining the pressure of a granular medium are for a stationary bed. However, Prante, as long ago as 1896, indicated a significant increase in the pressure in a granular medium when it starts to move. Later, Prante's observations were verified [53, 48–50], and corrections were made to the working formulas [53, 57] or new ones were proposed [48, 49] which described the pressure changes more fully.

Recent research has introduced substantial corrections into methods of determining the pressure in a granular medium, thereby significantly raising the reliability of the data obtained. A method of measuring the pressure has been worked out [58–60] and the construction of devices which would determine the value of the pressure in a granular medium at rest or in motion in any direction with minimal error (up to 5%) has been proposed.

Starting from the mechanism of pressure transmission proposed in [6], means of controlling the values of this pressure have been developed [61]. And a method permitting changes in the horizontal and vertical pressures has been worked out on the basis of the results obtained.

Investigations [69] devoted to the determination of the pressure in a funnel zone and the trajectories of boundary particles below the orifice are of definite interest.

An experimental verification of the proposed working equations showed good agreement.

The drag of structural elements and individual bodies of various geometry in a bulk solid flow have hardly been investigated at all. Gerasimov [62] has touched upon individual questions in passing, and a rather fuller treatment may be found in [63], where an attempt is made to propose working formulas to determine the drag of bodies of various shapes suspended in a granular flow.

The mechanics of granular media is in the process of development, as may be seen from the above survey. Previously, hypotheses that do not reflect the essence of the phenomenon have often been taken as basis, and this indubitably has resulted in incorrect conclusions. A significant number of investigations are of a particularly empirical nature, and as a result the recommendations made are applicable only under limited conditions.

Hence, we see from the state of the theory of the mechanics of granular media and the application of its laws in practice that the following issues are particularly pressing:

1) Extension of research to refine the basic hypothesis of the load transmission mechanism.

2) Study of the mechanical properties of bulk solid flows and, in particular, the particle packing density, the nature of internal friction in granular media, and the changes in internal friction as a function of the number of contacts and the state of the particle surface, the particle size, etc.

3) Investigation of the external friction mechanism, the determination of quantitative relations between the external friction and the physicomechanical properties of the granular medium and the enclosing walls.

4) Generalization of a large number of experiments in the area of the discharge of granular media from orifices, derivation of general discharge rate formulas taking account of the physicomechanical properties of the granular medium.

5) Study of the kinematics of granular medium flow under various conditions, discovery of laws governing the processes of granular media flow around various structural elements.

6) Investigation of the pressure exerted on the bottoms and walls of vessels of different design, refinement of existing working equations.

7) Investigation of the nature of the drag of structural elements and individual bodies of different geometry around which a granular medium flows, refinement of working equations and the coefficients taking account of the shape and orientation of the obstacles, surface states, and physicomechanical properties of the granular medium.

Thus, the increasing use of granular media in the national economy urgently demands a unified and purposeful effort on the part of the scientists working in this area.

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